Friction (Turbulent Drag Force)

- Friction always tends to slow down motion
- In the atmosphere, friction is strong only near the surface, and is negligible above the planetary boundary layer (about lowest 1 km of the atmosphere)

\[
\frac{F_x}{m} \bigg|_{TD} = -\kappa U
\]

\[
\frac{F_y}{m} \bigg|_{TD} = -\kappa V
\]

\(\kappa\) non-zero only close to the earth’s surface

Compare to Stull (9.12a,b)
The Horizontal Momentum Equations

• Putting the forces into Newton’s 2nd Law, we have

\[
\frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{F}}{m}\bigg|_{PG} + \frac{\vec{F}}{m}\bigg|_{CF} + \frac{\vec{F}}{m}\bigg|_{TD}
\]

• Separating them into x- and y- components, we have

\[
\frac{\Delta U}{\Delta t} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + fV - \kappa U
\]

\[
\frac{\Delta V}{\Delta t} = -\frac{1}{\rho} \frac{\Delta p}{\Delta y} - fU - \kappa V
\]

Compare to Stull (9.15a,b)

Note that if we start from rest (i.e., \(U = V = 0\)), both Coriolis force and turbulent drag force are identically zero. Hence the only force that can start motion is the pressure gradient force.
Why are winds nearly parallel to the isobars?

Above the level of friction, air initially at rest will accelerate until it flows parallel to the isobars at a steady speed with the pressure gradient force (PGF) balanced by the Coriolis force (CF). Wind blowing under these conditions is called geostrophic.
The isobars and contours on an upper-level chart are like the banks along a flowing stream. When they are widely spaced, the flow is weak; when they are narrowly spaced, the flow is stronger. The increase in winds on the chart results in a stronger Coriolis force \((CF)\), which balances a larger pressure gradient force \((PGF)\).
Geostrophic Wind (Stull p. 188-189)

• Above boundary layer, friction is negligible (i.e., can treat $\kappa \sim 0$)

• For straight line, unaccelerated flow, we have $\Delta U/\Delta t, \Delta V/\Delta t = 0$

• From horizontal momentum equations, we must have pressure gradient force balancing Coriolis force, i.e.

$$0 = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} + fV \quad 0 = -\frac{1}{\rho} \frac{\Delta p}{\Delta y} - fU$$

Stull (9.17)

– Which can be rewritten as:

$$U_g = -\frac{1}{\rho f} \frac{\Delta p}{\Delta y} \quad V_g = \frac{1}{\rho f} \frac{\Delta p}{\Delta x}$$

Stull (9.18)

See also Focus section on Ahrens p.216
Worked Example

• If the isobars are east-west oriented, and pressure increases northward at a rate of 1 hPa every 100 km
  – i. How strong is the pressure gradient force?
  – ii. What is the geostrophic wind (speed and direction)?
  – Use $\rho = 1 \text{ kg/m}^3$, and $f = 10^{-4} \text{ s}^{-1}$
• First transform into SI unit:
  – 1 hPa = 100 Pa, 100 km = 100000 m
  – i) Hence pressure gradient force:

\[
\frac{F_x}{m} = - \frac{1}{\rho} \frac{\Delta p}{\Delta x} = 0
\]

\[
\frac{F_y}{m} = - \frac{1}{\rho} \frac{\Delta p}{\Delta y} = - \frac{1}{1 \text{ kg/m}^3} \frac{100 \text{ Pa}}{100000 \text{ m}} = -10^{-3} \text{ ms}^{-2}
\]

– ii) Geostrophic wind

\[
U_g = - \frac{1}{\rho f} \frac{\Delta p}{\Delta y} = - \frac{1}{1 \times 10^{-4}} \frac{100}{100000} = -10 \text{ ms}^{-1}
\]

\[
V_g = \frac{1}{\rho f} \frac{\Delta p}{\Delta x} = \frac{1}{1 \times 10^{-4}} \cdot 0 = 0 \text{ ms}^{-1}
\]

– Hence wind direction is westward (easterly)
– See also example on Stull p.189
Geostrophic wind

In component form:

\[ U_g = -\frac{1}{\rho f} \frac{\Delta p}{\Delta y} \quad V_g = \frac{1}{\rho f} \frac{\Delta p}{\Delta x} \]

Alternatively:

Magnitude:

\[ |V_g| = \frac{1}{\rho f} \frac{\Delta p}{d} \]

Ahrens p. 216 equation (2)

Direction: Parallel to isobars, low pressure to the left in northern hemisphere
Classwork

• Compute the pressure gradient between points 5 and 6
  – PG = \( \Delta p/d \) (Ahrens p. 211)
    • \( \Delta p = 4 \text{ hPa} = 400 \text{ Pa} \)
    • d?
      – On map, scale is 1000 km : ?? mm
      – Distance for d: about ?? mm on map
      – Hence d \( \sim 800 \text{ km} = 800000 \text{ m} \)
      – PG = \( 400/800000 = 5 \times 10^{-4} \text{ Pa/m} \)
• Compute the magnitude of the geostrophic wind at point C (use the pressure gradient between points 5 and 6 to do that)
  – Geostrophic wind magnitude \( |V_g| = \frac{1}{\rho f} \frac{\Delta p}{d} \)

  • \( \Delta p/d \) from above
  • \( \rho = 1.2 \text{ kg/m}^3 \)
  • \( f = 2\Omega \sin(\text{latitude}) = 1.454 \times 10^{-4} \sin(45) = 1.028 \times 10^{-4} /\text{s} \)
  • \( |V_g| = 1/(1.2 \times 1.028 \times 10^{-4}) \times 400/800000 = 4.05 \text{ m/s} \)