Numerical Example

• An air parcel with mass of 1 kg rises adiabatically from sea level to an altitude of 3 km. What is its temperature change?
  – From the 1st law, $\Delta T = -g/c_p \Delta z + \Delta Q/m_{air}/c_p$
  – Here, $\Delta Q = 0$, hence $\Delta T = -g/c_p \Delta z = -10 \times 3 = -30$ K

• If the air parcel is moist, and 6 g of water vapor condenses during the ascent, what is the temperature change?
  – Latent heat is released when water vapor condenses
    • From above $\Delta Q = \Delta m \cdot L = 0.006 \times 2.5 \times 10^6 = 15000$ J
  – Hence $\Delta T = -10 \times 3 + 15000/1/1005 = -15$ K
Potential temperature

• We saw above that even without heat exchange with the environment, the temperature of an air parcel changes as it rises/subsides – hence temperature is not conserved

• One can show that for dry adiabatic processes, the quantity $\theta$ is conserved

$$ \theta = T \left( \frac{p_0}{p} \right)^\kappa $$

Stull (3.10)

  $\kappa = \frac{R_d}{c_p} = \frac{2}{7}$, $p_0 = 100000$ Pa

• $\theta$ is called the potential temperature

• If we know $\theta$, $T$ can be found by

$$ T = \theta \left( \frac{p}{p_0} \right)^\kappa $$
Numerical examples

• An air parcel at 900 hPa has temperature of 15 °C. What is its potential temperature?
  – First, write all quantities in SI unit
    • 900 hPa = 90000 Pa; 15 °C = (15 + 273.15) = 288.15 K
    – \( \theta = T(p_0/p)^\kappa \) = 288.15 x (100000/90000)^2/7 = 297 K
  
• If the air parcel is lifted adiabatically to 800 hPa, what will its temperature be?
  – Adiabatic process => \( \theta \) is conserved, hence \( \theta \) stays at 297 K
  
  – Given \( \theta \) and \( p \), we can find \( T \) by using \( T = \theta \ (p/p_0)^\kappa \)
    • Hence \( T = 297 \times (80000/100000)^2/7 = 278.7 \text{ K} = 5.5 \text{ °C} \)
Classwork Example

• The temperature of an air parcel at 700 hPa is 0 °C. The air parcel rises adiabatically to 300 hPa. What is its temperature there?
  – Adiabatic process – $\theta$ is conserved, hence need to find $\theta$ first
  – $P = 700$ hPa = 70000 Pa; $T = 0$ °C = 273.15 K
  – Hence $\theta = T \left( \frac{p_0}{p} \right)^{\kappa} = 273.15 \times \left( \frac{100000}{70000} \right)^{2/7}$
    $\quad = 302.45$ K
  – Given $\theta = 302.45$ K, $p = 300$ hPa = 30000 Pa,
    • $T = \theta \left( \frac{p}{p_0} \right)^{\kappa} = 302.45 \times \left( \frac{30000}{100000} \right)^{2/7} = 214.42$ K = -58.7 °C
Another example: see posted class notes

- The temperature of an air parcel at 500 hPa is –20 ºC. The air parcel subsides adiabatically to 800 hPa. What is its temperature there?
  - Adiabatic process – θ is conserved, hence need to find θ first
  - P = 500 hPa = 50000 Pa; T = -20 ºC = 253.15 K
  - Hence θ = T (p₀/p)κ = 253.15 x (100000/50000)²/⁷
    = 308.6 K
  - Given θ = 308.6 K, p = 800 hPa = 80000 Pa,
    - T = θ (p/p₀)κ = 308.6 x (80000/100000)²/⁷ = 289.5 K = 16.4 ºC
Review of Equations for Ch. 3

- Sensible heating of air \( \Delta Q = m_{\text{air}} c_p \Delta T \)
- Latent heat release \( \Delta Q = \Delta m \cdot L \)
- First law of thermodynamics \( \frac{\Delta Q}{m_{\text{air}}} = c_p \Delta T - \frac{\Delta P}{\rho} \)
- Dry adiabatic lapse rate \( \frac{\Delta T}{\Delta z}_{\text{adiabatic}} = -\frac{g}{c_p} \approx -10 \text{K} / \text{km} \)
- Potential temperature \( \theta = T \left( \frac{p_0}{p} \right)^\kappa \)
  - \( \theta \) is conserved for dry adiabatic lifting or subsiding air parcels
Seasonal Temperature Variations

• In the Northern Hemisphere, why is it hottest in late July, early August?
  – Is it because the earth is closest to the sun at that time?
    • No! Earth is closest to the sun in January
  – Is it because the NH gets the largest amount of solar energy at that time?
    • No! Largest amount of insolation in the NH occurs on June 21st, when the day is longest and the sun is most directly overhead
  – We will answer this question later
Solar intensity, defined as the energy per area, is one factor that governs earth's seasonal changes.

A sunlight beam that strikes at an angle is spread across a greater surface area, and is a less intense heat source than a beam impinging directly overhead.

animation
Daytime Warming

Solar radiation heats the earth’s surface. The atmosphere is heated from below by conduction and forced convection.

Winds create a forced convection of vertical mixing that diminishes steep temperature gradients.
• Question: During a day, when does the earth’s surface receive the maximum solar insolation?
• Question: When does maximum temperature occur?
• Why?
Temperature Lags

Earth's surface temperature is a balance between incoming solar radiation and outgoing terrestrial radiation.

Peak temperature lags after peak insolation because earth continues to warm until infrared radiation exceeds insolation.

\[ C \frac{dT}{dt} = J \]

\( J = \) net heating rate
• In a nutshell:
  – Maximum insolation at local noon:
    • At that time, incoming solar radiation exceeds heat loss from earth’s surface, hence surface temperature is still increasing
  – Maximum temperature occurs when rate of heat loss from earth’s surface is the same as rate of heat gained from solar insolation
    • That is, when net $J = 0$
    • This occurs later in the afternoon (usually)
• This applies also to seasonal variations:
  – Maximum insolation at summer solstice:
    • At that time, incoming solar radiation exceeds heat loss from earth’s surface, hence surface temperature is still increasing
  – Maximum temperature occurs when rate of heat loss from earth’s surface is the same as rate of heat gained from solar insolation
    • That is, when net $J = 0$
    • This occurs later in summer, usually around July or August